

GCE

Further Mathematics A

Y531/01: Pure Core

Advanced Subsidiary GCE

Mark Scheme for Autumn 2021

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in RM assessor	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	Meaning
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Question		Answer	Marks	AO	Guidance	
1	(a)	$8 - 2\lambda = -6 - 3\mu$ and $-11 + 5\lambda = 11 + \mu$ $8 - 2\lambda = -6 - 3\mu$ $-33 + 15\lambda = 33 + 3\mu$ $\Rightarrow -25 + 13\lambda = 27$ $\lambda = 4, \mu = -2$ $-2 + 3 \times 4 = 10$ and $8 - -2 = 10$ so they do intersect	B1 M1 A1 A1	1.1 a 1.1 1.1 2.4	Forming 2 correct equations in λ and μ . Could be $-2 + 3\lambda = 8 - \mu$ Attempt to solve (eg scaling one equation and adding or rewriting to a standard form for solution BC). Must reach an equation (possibly incorrect) with only one unknown. Both Checking for consistency in 3 rd equation and conclusion. Equation must be correct and both sides must be evaluated Allow eg $8 - 2 \times 4 = -6 - 3 \times -2$ $0 = 0$ Might see $\lambda = 4$ substituted into last equation and then μ being found with this.	Any two correct equations $-2\lambda + 3\mu = -14$ $5\lambda - \mu = 22$ $3\lambda + \mu = 10$ ie $-2 + 3 \times 4 = 8 - -2$ alone is not sufficient for A1, need to see both sides becoming 10 x: $8 - 2 \times 4 = 0$ & $-6 - 3 \times -2 = 0$ y: $-11 + 5 \times 4 = 9$ & $11 + -2 = 9$
	(b)	(0, 9, 10)	B1 [1]	1.1	Allow as vector	

Question			Answer	Marks	AO	Guidance	
2			$u = x + 1$	B1	3.1a	Attempt to expand using binomial. 4 terms. Substituting into equation. Allow if no “= 0” here. Must have an attempt at expanding $(u - 1)^3$ and $(u - 1)^2$ Must be an equation	Follow through on their $u = x + 1$ Follow through on their $u = x + 1$ For correct equation found using sums and products of roots allow SC2 (Method required was dictated in question) Only allocate marks using main scheme, or SC method
			$(u - 1)^3 = u^3 - 3u^2 + 3u - 1$ used in solution	M1	1.1		
			$2x^3 + 3x^2 - 2x + 5 = 0 \Rightarrow 2(u^3 - 3u^2 + 3u - 1) + 3(u^2 - 2u + 1) - 2(u - 1) + 5 = 0$	M1	1.1		
			$2u^3 - 3u^2 - 2u + 8 = 0$	A1	2.5		
				[4]			
Question			Answer	Marks	AO	Guidance	
3			$3 + 5i$ is a root	B1	1.2	Need to see statement that $3 + 5i$ is a root. Attempt to use the conjugate pair to derive a real quadratic	May happen at end of question May see $(3 + 5i)(3 - 5i) = 9 + 25 = 34$ and $(3 + 5i) + (3 - 5i) = 6$ instead of expansion
			Attempt to expand $(x - (3 + 5i))(x - (3 - 5i))$	M1	1.1		
			$= x^2 - 6x + 34$ so this must be a factor	A1	2.2a	Attempt to factorise or divide resulting in x^2 and one other term	NB: This question required detailed reasoning
			$x^4 - 7x^3 - 2x^2 + 218x - 1428 =$	M1	1.1		
			$(x^2 - 6x + 34)(x^2 + \dots x - 42)$ or $(x^2 - 6x + 34)(x^2 - x + \dots)$				
			$(x^2 - 6x + 34)(x^2 - x - 42)$	A1	1.1		
$(x^2 - x - 42) = (x - 7)(x + 6) \Rightarrow$ roots $-6, 7$ (and $3 + 5i$)	A1	1.1	3 + 5i may be mentioned as a root earlier in the solution				
				[6]			

Question			Answer	Marks	AO	Guidance	
4	(a)	(i)	Line drawn, perpendicular to line segment joining (0, -1) and (2,0)	M1	1.1	Line needs to have negative gradient with $ \text{gradient} > 1$ and to intersect the y axis at a positive value	If “shading out” is used then there needs to be an indication that the required region is below the line, such as “R” placed below line or “This region” written in etc.
			Region below line indicated as being the required region.	A1	1.1	Exact perpendicularity not needed, but should be approximately perpendicular.	
				[2]			
	(a)	(ii)	$m = -1/(1/2) = -2$ $4x + 2y - 3 = 0$	M1	1.1	Explicitly stated	Note must be in required form $ax+by+c=0$
				A1	1.1		
				[2]			
	(b)		Circle centre $(-1, 0)$ radius 3 or circle centre $(0, 2)$ radius 2. Both circles correct	M1	1.1	Radius can be implied by axis labels or tick-marks.	If M0A0 then SC1 for two circles with correct radii but centres $(1, 0)$ and $(0, -2)$
				A1	1.1		
			Correct region shaded or otherwise indicated	A1	1.1	Region inside circle with radius 3 but outside circle with radius 2.	
				[3]			

Question			Answer	Marks	AO	Guidance	
5	(a)		$\mathbf{AB} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix} = \begin{pmatrix} -\frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$ $\mathbf{BA} = \begin{pmatrix} \frac{5}{13} & -\frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{5}{13} & -\frac{12}{13} \\ -\frac{12}{13} & \frac{5}{13} \end{pmatrix} \neq \mathbf{AB}$ so matrix multiplication is not commutative	M1 A1 [2]	2.1 2.2a	BC. AB or BA correct. BC. Other multiplication correct and conclusion	Could see $\frac{1}{13} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 5 & -12 \\ 12 & 5 \end{pmatrix} = \frac{1}{13} \begin{pmatrix} -5 & 12 \\ 12 & 5 \end{pmatrix}$
	(b)		Rotation about O 67.4° anticlockwise	M1 A1 [2]	1.2 1.1	or 1.18 rads	1
	(c)		$(\mathbf{T}_B)^{-1}$ is a rotation about O by -67.4° anticlockwise (or 67.4° clockwise) So $\mathbf{B}^{-1} = \begin{pmatrix} \cos(-67.4^\circ) & -\sin(-67.4^\circ) \\ \sin(-67.4^\circ) & \cos(-67.4^\circ) \end{pmatrix}$ $= \begin{pmatrix} \frac{5}{13} & \frac{12}{13} \\ \frac{12}{13} & \frac{5}{13} \end{pmatrix}$	M1 A1 [2]	3.1a 1.1	Correct inverse of their rotation $\mathbf{T}_B$. or $\mathbf{B}^{-1} = \begin{pmatrix} 0.385 & 0.923 \\ -0.923 & 0.385 \end{pmatrix}$ (allow 0.384 for 0.385)	Could also be rotation of 292.6° anticlockwise NB: Question states “by considering the inverse transformation”. SC1 For correct inverse by other method.
	(d)		$\det \mathbf{B} = 1$ and $\det \mathbf{C} = -3$ So area of $N = 1 \times -3 \times 5 = 15$	M1 A1 [2]	3.1a 3.2a	Could find BC and then find $\det(\mathbf{BC}) = -3$ Area must be 15, do not allow -15 or ± 15	

Question			Answer	Marks	AO	Guidance	
6	(a)		$z = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 2 \times 25}}{2 \times 2}$	M1	2.1	Correct substitution into formula. If formula quoted allow one slip.	Or completing the square – one slip allowed.
			$z = \frac{5}{2} \pm \frac{5}{2}i$	A1	1.1	Allow $z = \frac{5 \pm 5i}{2}$ or equivalent fractions	NB: This question required detailed reasoning
			[2]				
	(b)		$3\omega - 2 = 5i + 2i\omega \Rightarrow 3\omega - 2i\omega = 2 + 5i$	M1	1.1	Expanding and rearranging	Must rearrange to isolate ω terms on one side and other terms on other side
			$(3 - 2i)\omega = 2 + 5i \Rightarrow \omega = \frac{2 + 5i}{3 - 2i}$	M1	1.1	Factorising and dividing by two term complex number	NB: This question required detailed reasoning
			$\omega = \frac{2 + 5i}{3 - 2i} \times \frac{3 + 2i}{3 + 2i} = \frac{6 + 4i + 15i - 10}{9 + 4}$	M1	2.1	Multiplying top and bottom by conjugate of bottom	
			$\omega = -\frac{4}{13} + \frac{19}{13}i$	A1	1.1		
			Alternative method				
			$\omega = a + bi \Rightarrow 3a + 3bi - 2 = 5i + 2ai - 2b$	M1		Assigning real and imaginary parts, to ω expanding and rearranging	
			$3a - 2 = -2b$ and $3b = 5 + 2a$	M1		Comparing real and imaginary parts	
			$9a - 6 + 10 + 4a = 0 \Rightarrow a = -\frac{4}{13}$	M1		Using valid algebra to eliminate one unknown and finding the other	
			$\Rightarrow b = \frac{19}{13} \Rightarrow \omega = -\frac{4}{13} + \frac{19}{13}i$	A1			
				[4]			

Question			Answer	Marks	AO	Guidance	
8	(a)		$(t-1)(6-t(2-2t))$ $-(t-1)((1-t)-t(2-2t))$ $+(t-1)((1-t)(2-2t)-6(2-2t))$	M1	1.1	Correct process for expanding determinant.	Fully expanded form: $2t^3 + 7t^2 - 14t + 5$
			$(t-1)[(6-t(2-2t)) - ((1-t)-t(2-2t))$ $+((1-t)(2-2t)-6(2-2t))]$	M1	1.1	Bringing $(t-1)$ or $(t+5)$ or $(2t-1)$ out as factor of the entire expression	Factors may appear BC from no working
			$(t-1)(6-2t+2t^2-1+t+2t-2t^2+2-4t$ $+2t^2-12+12t)$ $= (t-1)(2t^2+9t-5)$ $= (t-1)(2t-1)(t+5)$	A1	1.1		
				[3]			
	(b)		$-5, \frac{1}{2}, 1$	B1	1.1	FT their complete factorisation of determinant into 3 linear factors.	
				[1]			
	(c)		$t = b^2 + 2$	M1	2.1	So that the system is $\mathbf{Ar} = \mathbf{c}$	
			and so $t \geq 2$ so cannot be $-5, \frac{1}{2}$ or 1 therefore $\mathbf{A}^{-1}$ will exist (for all values of b) and so there will be a unique solution to the system for all values of b .	A1	2.4	Complete reasoning must be seen for A1.	Could test $t = 1, \frac{1}{2}, -5$ in $b^2 = t - 2$, and show that these do not give real values of b
				[2]			

Question			Answer	Marks	AO	Guidance	
9	(a)		$\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 3 \\ -16 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \\ -21 \end{pmatrix} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix}$ $\begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ s \\ t \end{pmatrix} = 0$ $-4 - 2s + 5t = 0$ $\Rightarrow 2s = 5t - 4$ $\Rightarrow s = 2.5t - 2$	M1 M1 A1 [3]	2.1 1.1 2.1	Attempt to find the direction vector of the tunnel. Any non-zero multiple. Use of $\overrightarrow{PQ} \cdot \mathbf{b} = 0$ in the solution. AG. Some intermediate work must be seen.	
	(b)		$\mathbf{M} = \frac{1}{2} \left(\begin{pmatrix} -1 \\ 3 \\ -16 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \\ -21 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ s \\ t \end{pmatrix} \text{ when } z = 0$ $\Rightarrow -18.5 + \lambda t = 0$ $\Rightarrow \lambda = \frac{18.5}{t} \text{ (so } c = 18.5)$	B1 M1 A1 [3]	1.1 3.4 1.1	Position vector (or co-ordinates) of mid-point found Using $z = 0$ and the equation of the line to find a 'horizontal' relationship between λ and t . 	Condone errors in, or omission of, x and y components. NB: Question can be answered just by considering the z coordinate. If done correctly and M1 A1 gained also allow B1 as implied.

Question		Answer	Marks	AO	Guidance	
	(c)	So we need to minimise $\left \frac{18.5}{t} \begin{pmatrix} 1 \\ 2.5t-2 \\ t \end{pmatrix} \right$ $(y =) \frac{1369}{4t^2} (1 + (2.5t-2)^2 + t^2)$ $= \frac{1369}{4} (7.25 - 10t^{-1} + 5t^{-2})$ So to minimise set $\frac{dy}{dt} = \frac{1369}{4} (10t^{-2} - 10t^{-3}) = 0$ $10t^{-2} - 10t^{-3} = 0 \Rightarrow t = 1$ So length of shaft = $\left 18.5 \begin{pmatrix} 1 \\ 0.5 \\ 1 \end{pmatrix} \right$ or $\sqrt{\frac{1369}{4} (7.25 - 10 \times 1^{-1} + 5 \times 1^{-2})}$ oe $= 18.5 \times 1.5 = 27.75$	M1 M1* dep M1* A1 M1 A1	3.3 1.1 3.1 a 2.2a 3.4 1.1	Stating or implying that the length of the shaft is given by $\lambda \mathbf{b}$ and using their λ / t relationship to reduce length of shaft to a form with only one variable. Finding expression for (squared) length of their vector Correct method for minimisation of (squared) length of their vector (eg differentiating and setting to 0) Substituting their t into their form for length of shaft	Or eg $\left \frac{18.5}{0.4s+0.8} \begin{pmatrix} 1 \\ s \\ 0.4s+0.8 \end{pmatrix} \right$ May see $\frac{37}{2} (7.25 - 10t^{-1} + 5t^{-2})^{\frac{1}{2}}$ Or $\frac{39701}{16} - \frac{6845}{2} t^{-1} + \frac{6845}{4} t^{-2}$ oe Or attempt to complete the square in t^{-1}. $y = \frac{1369}{4} (5(t^{-1} - 1)^2 + 2.25)$ So min when $t^{-1} - 1 = 0, t = 1$

Question			Answer	Marks	AO	Guidance	
			Alternate method: $\mathbf{a} = \frac{1}{2} \left(\begin{pmatrix} -1 \\ 3 \\ -16 \end{pmatrix} + \begin{pmatrix} 3 \\ 5 \\ -21 \end{pmatrix} \right) = \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} = 2 \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ $(k)\mathbf{b} = \begin{pmatrix} -2 \\ 4 \\ 0 \end{pmatrix} \times \begin{pmatrix} -4 \\ -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 20 \\ 10 \\ 20 \end{pmatrix} = 20 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ Need $-18.5 + \lambda = 0 \Rightarrow \lambda = 18.5$ So length of shaft = $\left 18.5 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right$ $= 18.5 \times 3/2 = 27.75$	B1 M1 M1 M1 M1 A1		Attempt to find normal to vertical plane containing tunnel Attempt to find (multiple of) b by crossing their n with direction vector of tunnel. Using $z = 0$ to find λ	May see multiple of b used eg $-18.5 + 2\lambda = 0$ May see eg $\mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix} + 9.25 \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 19.5 \\ 13.25 \\ 0 \end{pmatrix}$ and then $\begin{pmatrix} 19.5 \\ 13.25 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -18.5 \end{pmatrix}$
				[6]			
	(d)		So b is not parallel to the z -axis so the ventilation shaft does not go straight down.	B1 [1]	3.2a	Shaft not vertical	

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